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IN SITU DETERMINATION OF ELASTIC STIFFNESS CONSTANTS OF THICK COMPOSITES

Xiaorong Lu, Kwang Yul Kim & Wolfgang Sachse*

Department of Theoretical and Applied Mechanics, Cornell University, Ithaca, New York 14853-1503, USA

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Abstract

We summarize in this paper some of the challenges of making ultrasonic measurements in thick composites and, in particular, in determining non-destructively the matrix of elastic constants of such materials. The procedure relies on the point-source/point-receiver measurement technique by which various wave modes are simultaneously generated over a broad angular spectrum in a test specimen. A procedure is described for analyzing the measured wavespeeds to determine the elastic symmetry of a composite test specimen and its matrix of elastic moduli. © 1997 Published by Elsevier Science Limited

Keywords: B. mechanical properties; C. anisotropy; D. non-destructive testing; D. ultrasonics; elastic properties

1 INTRODUCTION

Composite materials can be designed and fabricated to produce parts of complex geometries that possess specific physical, mechanical and electromagnetic properties and at the same time exhibit good corrosion and wear resistance, with optimum strength- and stiffness-to-weight ratios along particular directions. Our interest here is in so-called thick composites, which are materials whose thickness ranges from 25·4 to 203 mm (1 to 8 in). Thick composites have become essential for the fabrication of a number of aerospace structural components and they are also being increasingly considered for naval systems such as submersible vessels and surface ship structural elements. To ensure the integrity of such structures, appropriate non-destructive techniques must be used. A summary of the current approaches which are based on ultrasonic measurements has been given in Ref. 1. In this paper we describe a non-destructive procedure that utilizes ultrasonic wavespeed measurements to determine the elastic properties of a thick composite, expressed in terms of the matrix of elastic constants of the composite.

* To whom correspondence should be addressed.

2 NON-DESTRUCTIVE CHARACTERIZATION OF THICK COMPOSITES

The characterization of a thick composite might be carried out during fabrication, after assembly, or in service. As a result, there are often significant constraints under which the measurements need to be made. Non-destructive techniques for monitoring and controlling the processes are needed during the fabrication of composite materials and structures to monitor the ply stacking sequence, the fiber/resin ratio and the winding characteristics when appropriate. Techniques are also sought to detect inclusions and contamination, control the cure process, determine and control the dimensional characteristics including the material edges and, in some cases, determine the integrity and strength of any adhesive bonds. But in addition, the inspection of thick composite materials or structures after assembly or in service will need to: (1) characterize the spatial variation of the material's composition, configuration (ply lay-up, thicknesses, etc.), voids, inclusions and porosity; (2) determine the magnitude and extent of residual stresses; (3) determine the presence and number of fiber/matrix disbonds and/or failures; and (4) detect the presence and extent of matrix cracking.

Ultrasonic techniques, which utilize the propagation of elastic waves through a material, form the basis of a number of procedures for non-destructive determination of the elastic properties and detection of material inhomogeneities and flaws in thick composites. It would be expected that such measurements can form a useful tool for carrying out several of the above-mentioned materials characterization tasks. In this paper, we focus on the non-destructive characterization of the anisotropic stiffness properties of thick

composites. These are expected to be an indicator, to varying sensitivity, of the characteristics of the composite described in the previous paragraph.

The conventional approach for determining the elastic moduli of an anisotropic material relies on pulsed ultrasonic measurements made on samples cut from the material such that the equations relating elastic moduli and ultrasonic wavespeeds are easily inverted. A non-destructive characterization procedure, however, requires the *in situ* determination of the elastic stiffness properties of a test section which may be part of a structure. In order to apply ultrasonic measurements in composite specimens and, in particular, to make measurements along arbitrary directions in a thick composite, a number of material- and geometry-related measurement challenges must be overcome. These are listed in Table 1. We shall describe in Section 4 a procedure for recovering the elastic constants of a thick composite structure that utilizes ultrasonic signals propagating along arbitrary directions in an unprepared specimen, without requiring any cutting or other specimen preparation.

3 POINT-SOURCE/POINT-RECEIVER ULTRASONIC SYSTEM

The ultrasonic point-source/point-receiver (PS/PR) measurement system, which has been developed over the past decade, has the potential for meeting many of the measurement challenges listed in Table 1. Under certain operating conditions, such a system can also be accurately modeled and it can therefore be used to make quantitative ultrasonic measurements in materials.[2–5] The basis of the technique is a small-aperture source and receiver in which knowledge of the wave propagation is used to interpret the detected ultrasonic signals. In this approach, one trades added complexity in extracting material properties from ultrasonic wavefield data for simplicity in specimen preparation and data collection, which leads to reliability in the measurement procedure.

The PS/PR technique has shown great promise in several materials inspection and characterization applications because it exhibits a number of distinct advantages. Except in special cases, a point source generates both longitudinal and shear waves in a specimen; therefore information about each of these wave modes can, in principle, be extracted from each waveform. Further, since PS/PR signals are simultaneously propagated in a wide range of directions in a specimen, one can use an array of sensors or scan either the source or the receiver over the specimen surface to determine the directional dependence of the speeds of propagation and amplitudes of various wave modes in a material. When a repetitive source is used, a number of signals measured at adjacent source/receiver configurations can be stacked together to obtain a so-called scan image which represents the detailed spatial and temporal characteristics of the elastic wave field in a material that can be directly related to the material's anisotropy and macrostructure.[6,7]

The requirements and operational characteristics of a number of sources and receivers that can be used in a PS/PR measurement system have previously been described.[8] For example, if a step-type source time function is sought to generate signals possessing a broad frequency spectrum, the point source may be a normal force which can be obtained from the fracture of a capillary or a focused, pulsed laser beam of sufficient energy that it results in material ablation at

Table 1. Materials- and geometry-related measurement challenges for thick composites

S/N ratio, SNR	Because of frequency-dependent absorption, the *signal-to-noise* ratio of the detected ultrasonic signals is limited. This leads to difficulties in detecting material flaws and inhomogeneities and also for extracting accurate wave arrival-time and signal-amplitude information from the band-limited ultrasonic signals.
Elastic anisotropy	In arbitrary directions in an anisotropic composite material, three possible bulk wave modes are generated, one *quasi*-longitudinal mode and two *quasi*-transverse wave modes whose speed of propagation depends on the propagation direction in the specimen. Also, along arbitrary directions in such a specimen, the flux of ultrasonic energy usually propagates in a direction that differs from the direction of the wave normal. That is, the *group velocity* and the *phase velocity* in a specimen in an arbitrary direction are not equi-valued. Not only do these phenomena complicate flaw location and sizing procedures, but they add a level of complexity in extracting the matrix of elastic constants from ultrasonic group velocity data.
Wave distortion	The viscosity and heterogeneities in a composite specimen may lead to wave dispersion, speckle, as well as non-linear effects. Correspondingly, an ultrasonic waveform may change its shape as it propagates through a composite material which complicates wave arrival-time measurements as well as conventional signal-processing procedures that rely on amplitude or spectral magnitude measurements.
Complex geometries	The proposed applications of thick composites generally result in test specimens that are more than 25 mm thick and which span large areas. They are often curved or possess otherwise complex-shaped geometries.
Structure surfaces	The surfaces of the composite structures are often non-planar and non-parallel. They may also possess complex curvatures and exhibit a surface roughness whose characteristic dimension is of the order of the wavelength of the ultrasonic signals used to probe the material.
Inspectability	Structural considerations often dictate that there be a minimum number of joints. The result is that access to internal regions on a structure for measurements is limited.

the specimen surface. The generated ultrasonic signal in the material has a strong first arrival, longitudinal wave mode (or *P* wave) amplitude. A horizontal double force is realized when the pulsed laser beam is operating in the thermoelastic regime. In that case, a strong shear wave signal is generated in the specimen.

For the measurements presented in this paper, the point source was the fracture of a glass capillary ~0·1–0·25 mm in diameter and the point detector was a miniature piezoelectric transducer 1·3 mm in aperture. The source was activated on top of a 70 μm thick, piezoelectric, poly(vinylidene fluoride) (PVDF) polymer film and the resulting step unloading provided a repeatable, fast fall-time signal in synchronization with the excitation. The signal detected by the piezoelectric sensor was input to a preamplifier whose bandwidth ranged from 2 kHz to 2 MHz. When a pure transverse wave mode was required, a piezoelectric, contact shear transducer was used. In all cases, the detected signals were recorded with a two-channel, 10-bit waveform recorder operating at sampling rates of up to 30 MHz and controlled by a PC.

To date, ultrasonic PS/PR measurements have been made in a broad variety of polymer-fiber and ceramic-fiber/ceramic-matrix composite specimens of thickness ranging from 120 μm to 139·7 mm (5·5 in), see Refs. 9–11. In every case, signals possessing excellent signal-to-noise ratio (*SNR*) are detected even when the materials exhibit high damping characteristics that make conventional ultrasonic measurements difficult or even impossible.

Figure 1(a) illustrates the waveforms detected on the surface of a 25·4 mm (1 in) thick glass-fiber composite specimen. The first arrival signal, corresponding to the longitudinal wave, generates a transducer output signal of amplitude about 25 mV. It is seen that the time interval between the synchronization signal, which coincides with the instant of source activation, and the first-arrival signal detected at the receiver location is approximately 9·6 μs; this corresponds to a longitudinal group velocity normal to the specimen surface of about 2·65 mm μs^{-1} in this material. The step unloading source signal is broadband with frequencies greater than 10 MHz. The normalized frequency magnitude spectrum of the detected output signal, shown in Fig. 1(b), exhibits a peak at about 40 kHz. Despite the strong absorbence of frequencies higher than 150 kHz, the detected signal exhibits a good *SNR* and its arrival can easily be determined to within about 50 ns.

To illustrate the capability of the PS/PR technique in making measurements in ultra-thick composites, we show a sample testing configuration in Fig. 2. Also shown in this figure is the detected ultrasonic signal. Although in this specimen the amplitude of the first arrival is only about 2·0 mV, the *SNR* is adequate to permit identification of the first-arrival signal in the detected transducer signal. The travel time through the 139·7 mm (5·5 in) thick specimen is 52·5 μs, corresponding to a longitudinal wavespeed of 2·66 mm μs^{-1} in this direction.

4 RECOVERY OF ELASTIC STIFFNESSES

Procedures for recovering the complete matrix of elastic constants of an anisotropic composite specimen from wavespeed measurements made along non-principal directions have been the focus of much recent research. A number of wavespeed data inversion procedures have been developed. Here we give only a brief review and a summary of the approach we have implemented.

Analytical relationships between the elastic constants of an elastically anisotropic solid and the

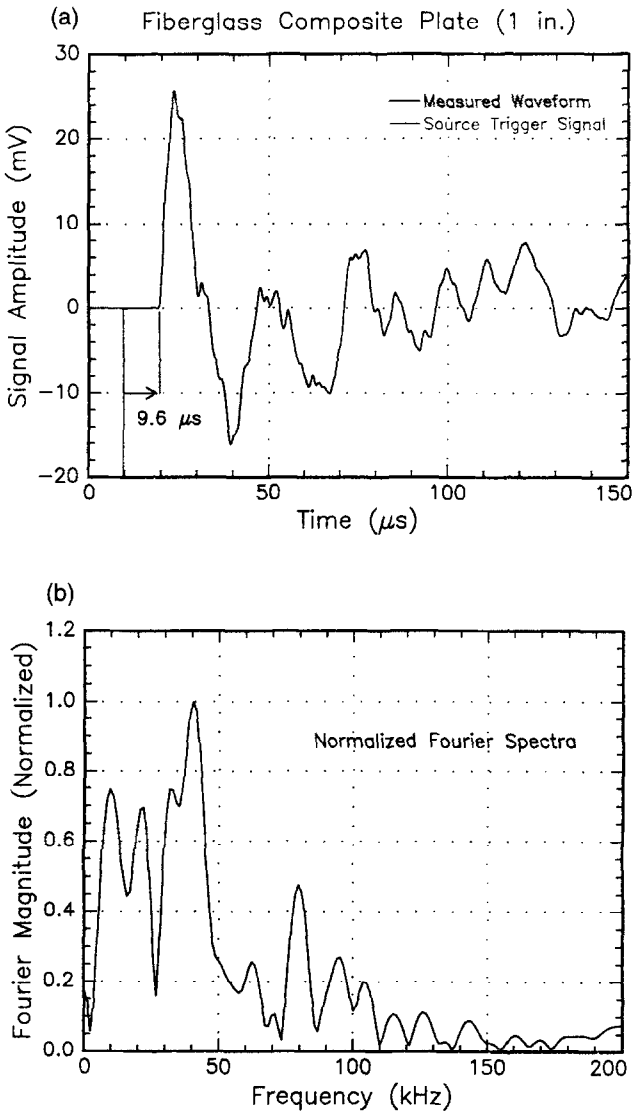

Fig. 1. Ultrasonic PS/PR measurement in a 25·4 mm (1 in) thick glass-fiber composite plate. (a) Detected waveform. Time zero coincides with the onset of excitation; longitudinal wavespeed result: 2·65 mm μs^{-1}. (b) Fourier magnitude spectrum.

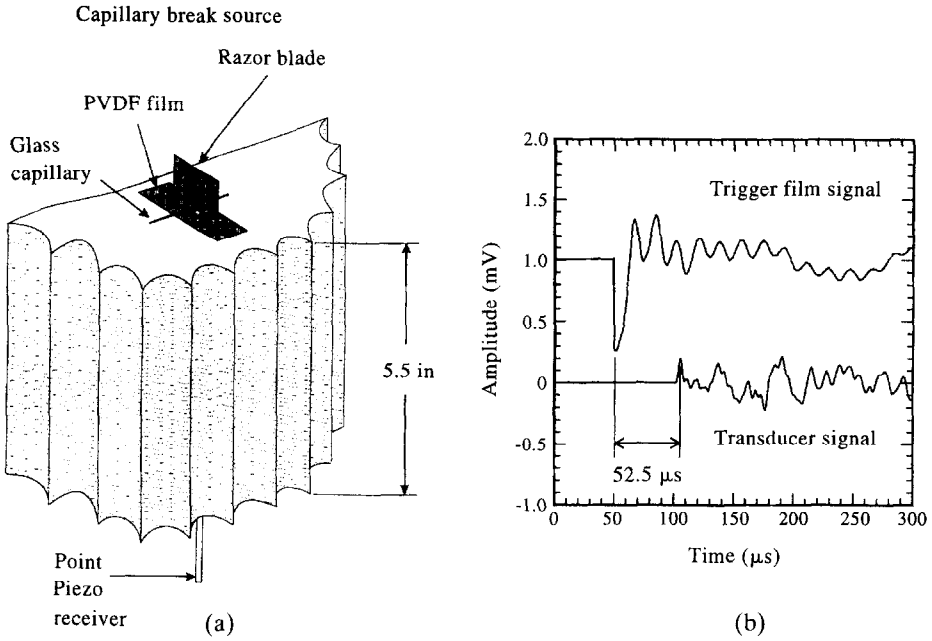

Fig. 2. Ultrasonic PS/PR measurement in an ultra-thick composite. (a) Specimen test geometry; glass capillary fracture source; 1·3 mm aperture piezoelectric transducer sensor. (b) Detected waveform. Longitudinal wavespeed result: 2·66 mm μs^{-1}.

acoustic phase velocities propagating in a general direction in the material are found from the solution of Christoffel's equation.[12,13] Thus, if one assumes that the measured speeds of propagation correspond, at least approximately, to the phase velocities, an optimized fitting procedure can be used to recover the full set of elastic constants of the material. This has been demonstrated for cubic, transversely isotropic and orthorhombic materials.[14,15]

From PS/PR measurements on a thick composite, one determines the time taken for a particular bulk wave signal (longitudinal or shear) to propagate from the source to the receiver point. The ratio of the distance divided by the travel time corresponds to the speed of propagation or group velocity of the wave mode associated with the signal through the material. The group velocity corresponds to the propagation of energy through a material. For waves propagating along symmetry directions in an anisotropic material, the magnitude of the group velocity equals the phase velocity. But along arbitrary directions, the group and phase velocities are not equal, in either magnitude or direction; that is, the direction of energy propagation and the wave normal do not coincide. This complicates the wavespeed inversion procedure.[16]

Until recently, there existed no explicit analytical functions relating the elastic constants to the group velocities along arbitrary directions in an anisotropic solid. Kim[17] has derived such relationships for waves propagating along arbitrary directions in symmetry planes of materials that are elastically orthotropic or of higher symmetry. We shall show later that the thick composite specimen, which is of particular interest here, can be represented as being transversely isotropic. For this reason, in the following paragraphs we shall describe the wavespeed inversion procedure specific to a material possessing this symmetry.

Transversely isotropic materials are characterized by five independent elastic stiffness constants. If X_3 is denoted as the principal symmetry direction or the axis of transverse isotropy, these constants written in Voigt notation, $C_{\alpha\beta}$, are:

$$C_{11}(=C_{22}), C_{33}, C_{12}, C_{13}(=C_{23}) \text{ and } C_{44}(=C_{55}) \quad (1)$$

In addition, the following relationship holds for a transversely isotropic medium:

$$C_{66} = (C_{11} - C_{12})/2 \quad (2)$$

For waves propagating along the principal symmetry direction X_3 in the specimen, the phase and group velocities are identical. The longitudinal wave, traveling with speed V_L along this direction, is directly related to the elastic constant C_{33} according to:

$$C_{33} = \rho V_L^2 \quad (3)$$

Similarly, the transverse wave mode propagating with speed V_T along this direction yields the value of C_{44}:

$$C_{44} = \rho V_T^2 \quad (4)$$

It is seen that use of group velocity data of longitudinal and shear waves that have been measured along the principal direction X_3 yields two of the five independent elastic stiffness constants of the material.

However, for waves propagating along directions other than the principal symmetry direction, the wave normal n and the direction of the group velocity V_g are usually not identical. As shown in Fig. 3, we denote

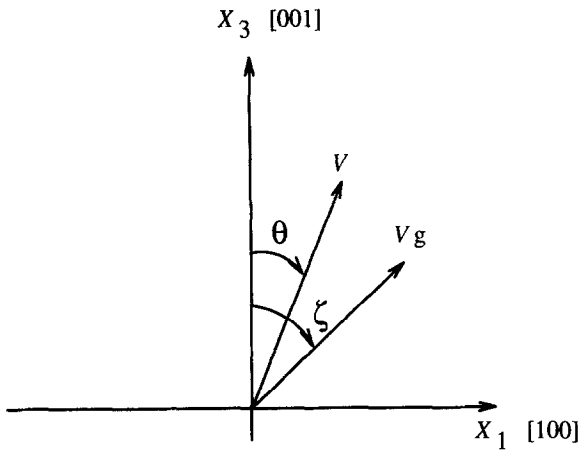

Fig. 3. Directions associated with the group velocity, V_g, and the phase velocity, V.

their respective directions by the angles θ and ζ, respectively, measured relative to the X_3 direction for the case in which the wave is propagating on the X_1X_3 plane. The group velocity of pure transverse waves possessing shear-horizontal (SH) polarization is related to the angle ζ by:

$$\frac{1}{\rho V_g^2} = \frac{\sin^2\zeta}{C_{66}} + \frac{\cos^2\zeta}{C_{44}} \tag{5}$$

By using this equation and the previously determined value of C_{44}, the elastic constant C_{66} can be found from the group velocity of the pure transverse wave mode measured along an arbitrary angle of ζ relative to the X_3 direction, with the equation

$$C_{66} = \frac{\rho V_g^2 C_{44}\sin^2\zeta}{C_{44} - \rho V_g^2\cos^2\zeta} \tag{6}$$

If one were permitted to cut the sample such that faces normal to the X_1 direction could be accessed, then it would be a straightforward procedure to obtain the value of C_{11} from the measured group velocity of the longitudinal wave along that direction simply by using an equation analogous to eqn (3). The use of shear waves whose polarizations are perpendicular to the axis of transverse isotropy can be used to evaluate the constant C_{12}. Another specimen cut at 45° to the X_3 axis would be required to permit determination of the elastic constant C_{13}.[18] However, since cutting of the test specimen is to be avoided, we require an alternative procedure that utilizes group velocity data measured along various directions ζ on the X_1X_3 symmetry plane. To do this, the group velocity data are first converted to phase velocity data which can then be processed with a least-squares optimization procedure to recover the elastic constants C_{11} (or C_{12}) and C_{13}.

To obtain group velocity data at various angles ζ to the X_3 axis, the source is moved to a number of locations along a straight line passing through the epicenter point on the surface of the specimen opposite that on which the point detector is mounted. The measured group velocity data are then fit to the polynomial function:

$$V_g(\zeta) = \sum_{n=0}^{N} a_n\zeta^n \tag{7}$$

Not only does this approach minimize the effects of random measurement errors but it also leads to a function which can be differentiated in closed form so that it can be input into the following equation to permit evaluation of the magnitude of the phase velocity:

$$V(\zeta) = \frac{[V_g(\zeta)]^2}{\sqrt{V_g^2 + (dV_g/d\zeta)^2}} \tag{8}$$

The group velocity propagating in direction ζ and the direction of the wave normal specified by the angle θ satisfy the equation:

$$\theta(\zeta) = \zeta - \tan^{-1}\left(\frac{1}{V_g}\frac{dV_g}{d\zeta}\right) \tag{9}$$

The two foregoing equations permit conversion of the measured group velocity data $V_g(\zeta)$, by first transforming it into two parametric equations for the phase velocity and the wave normal, i.e. $V(\zeta)$ and $\theta(\zeta)$, respectively. These equations are then combined into one equation to give $V(\theta)$, the phase velocity as a function of wave normal θ. Additional details regarding eqns (7)–(9) and the conversion between phase and group velocities have been given by Kim *et al.*[17,19] While the foregoing forms the basis of a general approach for recovering the complete matrix of elastic constants, one can often pursue a simplified approach to determine the complete matrix of elastic constants.

As mentioned previously, eqns (3)–(5) are used to determine the subset of elastic stiffness constants C_{33}, C_{44} and C_{66}. For a material of transverse isotropy, the remaining constants, C_{11} (or C_{12}) and C_{13}, can be found with the following procedure which relies on phase velocity data of the quasi-longitudinal wave mode calculated from the group velocity data measured on one symmetry plane, such as the plane X_1X_3.

For simplicity, we define the following quantities:

$$C_{11\pm} \equiv C_{11} \pm C_{55},\ C_{33\pm} \equiv C_{33} \pm C_{55} \quad \text{and} \quad C_{13\pm} \equiv C_{13} \pm C_{55} \tag{10}$$

Expressed in terms of these quantities, the phase velocity of the quasi-longitudinal wave as a function of wave normal on the X_1X_3 plane is given by:[12,13]

$$2\rho V_{QL}^2 = C_{11+}\sin^2\theta + C_{33+}\cos^2\theta \pm \sqrt{(C_{11-}\sin^2\theta - C_{33-}\cos^2\theta)^2 + 4C_{13+}^2\sin^2\theta\cos^2\theta} \quad (11)$$

Substituting the previously evaluated constants C_{33} and C_{55} (which is equal to C_{44} for a transversely isotropic material) into eqn (11), the remaining elastic constants, C_{11} and C_{13}, can be found by a least-squares fitting of the quasi-longitudinal phase velocity data, $V_{QL}(\theta)$, into the target equation [eqn (11)]. The remaining, unknown elastic constant, C_{12}, is then determined by substituting the values obtained for C_{11} and C_{66} into eqn (2).

5 MEASUREMENTS

5.1 Description of the specimen

We demonstrate the recovery procedure for the elastic stiffness constants on a large, nearly circular, thick composite plate fabricated from a glass-reinforced plastic material consisting of a high-grade, toughened, isophthalic marine polyester resin with E-glass fiber reinforcement. The average density of the material was 1·766 g cm^{-3}. The geometry of the plate is shown in Fig. 4. The plate-like specimen was nominally 633 mm in diameter with a flat spot on its circumference that extended for 320 mm. In addition, the plate had a 40 mm diameter hole at its center. One surface of the plate was smooth to the touch while the other was rough. The plate thickness was not uniform, particularly near the flat edge.

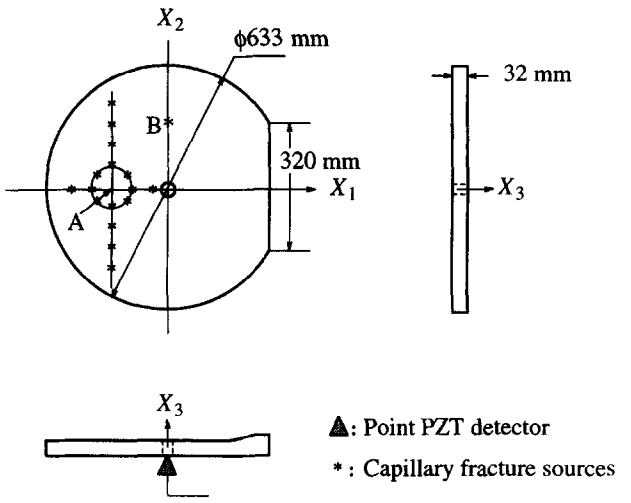

Fig. 4. Specimen geometry showing source and detector locations.

PS/PR measurements require that the distance between the source and receiver points be known accurately, since this distance divided by the travel time is the value of the group velocity in that direction in the material. The measurements were made with the source activated on one surface of the plate at points along a straight line passing through epicenter. In Fig. 4 are shown the points 'A' or 'B' to indicate the epicenter points for two different measurement series. Since the distance for points along a measurement line is known precisely, only the plate thickness along the line needs to be determined. In order to determine the thickness of the plate, the rough surface was scanned with a digital displacement gage to map out the relative surface thickness variation along the measurement line to a resolution of 2·5 μm. Since the thickness along the edge of the plate can be measured precisely, the thickness at an interior point of the plate can be determined from the relative heights of the interior point and the edge. The assumption made is that the roughness of the smooth surface is negligible compared with the plate thickness. One of the measured relative surface thickness curves is shown in Fig. 5. It corresponds to the region over which PS/PR measurements were made. Such measurements provide a quantitative measure of the surface topography along the measured line.

In fact, one does not need to know the thickness of the plate at all the interior measurement points if the detector is mounted on the rough surface of the plate and the source is activated on the smooth side. In this case, as depicted in Fig. 6(a), the wave propagation distances are independent of the surface roughness. In contrast, when the source is activated on the rough side and the detector is attached on the smooth side, the thickness variation must be included at each measurement point, as depicted in Fig. 6(b). For the former, one needs to only know the thickness of the plate at the epicenter point, i.e. the position directly

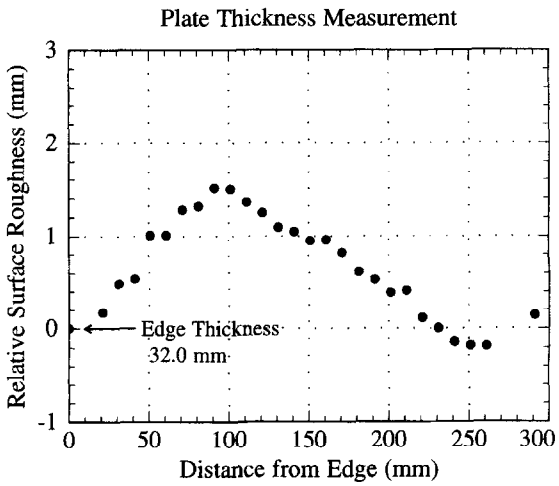

Fig. 5. Thickness measurement of the composite plate.

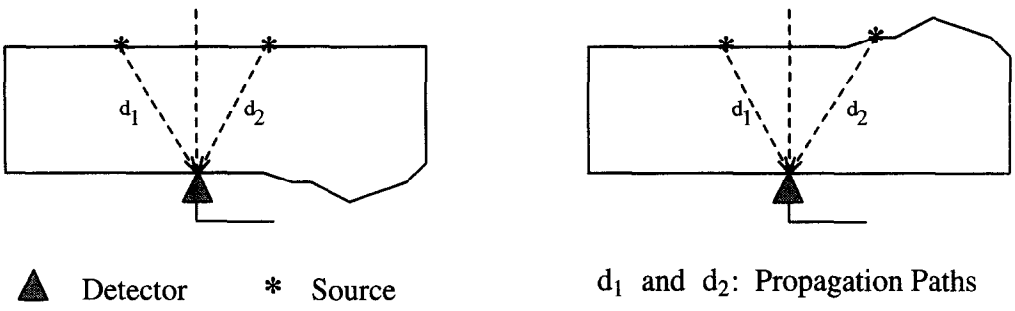

Fig. 6. Different configurations of source and detector locations.

above the detector. From the measured surface roughness curve shown in Fig. 6, the thickness at the detector site was 32·9 mm.

5.2 Identification of anisotropy

To determine the anisotropy of the composite plate, two sets of experiments were performed. The purpose of the first was the determination of the anisotropy over a small local area of the plate. To do this, the capillary fracture source was activated at points along a circle of diameter 40 mm (shown in Fig. 4) on the top (smooth) surface and the point detector for the longitudinal wave was located at the center of the circle on the bottom surface. Along the circle, the wave propagation distance is identical for all locations of the source so that the measured group velocities are directly related to the wave travel times. Figure 7 shows the measured wave travel times of the first-arrival signal for different points on the circle. The source locations are represented in terms of their angular location relative to the horizontal axis X_1 shown in Fig. 4.

The data in Fig. 7 show that the measured travel times are identical within the range of experimental error. To make certain that the plate is also homogeneous over a broader area, additional measurements were made along circles which were at radial distances of 6 cm and 8 cm from the epicenter. Two epicenter points were chosen, denoted as 'A' and 'B' in Fig. 4. The results obtained around each of these points are shown in Fig. 8. Each value represents the averaged value of two or three separate measurements. In view of the experimental errors and the homogeneity of the plate, these data demonstrate that the plate may be either isotropic or transversely isotropic with respect to the X_3 axis.

5.3 Identification of pure transverse wave arrival

To determine the shear elastic stiffness constants C_{44} and C_{66}, the wave arrival time of the pure transverse ('PT') wave mode is required. As mentioned earlier, this wave mode was generated by using two broadband piezoelectric contact shear transducers that were 6·35 mm in aperture and were operated in the 'pitch–catch' mode. While transducers with such apertures are really not point transducers, when used to make measurements over a small range of angles to direct transmission in thick and highly absorptive

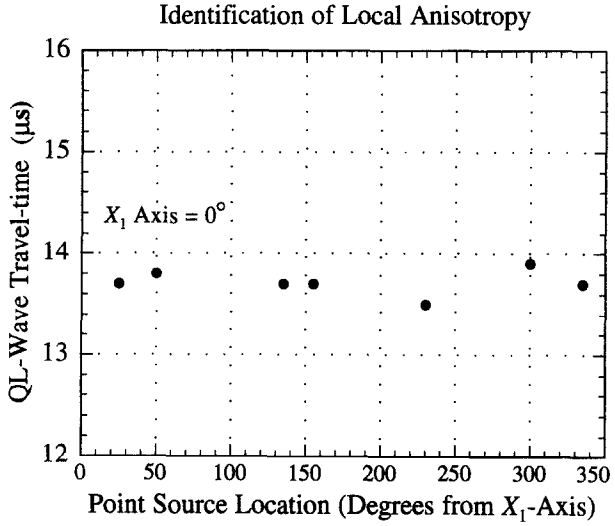

Fig. 7. Check of the local anisotropy.

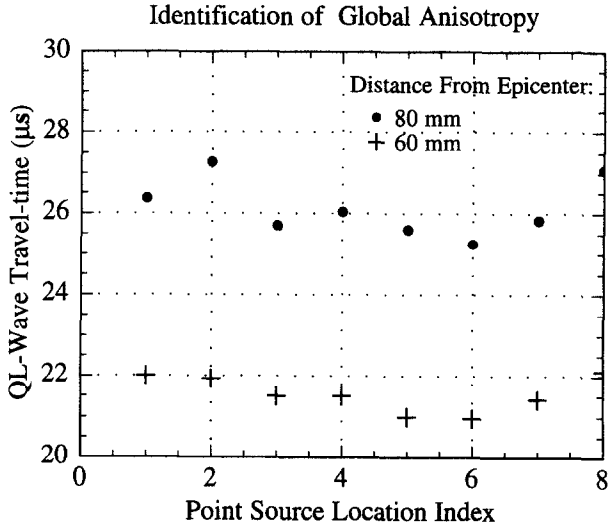

Fig. 8. Identification of specimen global anisotropy.

composites for which the higher frequency components of the signal are absorbed, such transducers can reasonably well resemble the operation of point-like devices. The excitation was an electrical impulse obtained from an ultrasonic pulse generator (Panametrics 5055 P/R). Both the source and the detected signals were recorded in a waveform recorder operating at a sampling rate of 30 MHz. The source signal indicates the time of excitation, thus again providing a convenient reference for determination of the absolute arrival times of the pure transverse wave propagating from the source to the detector.

For most materials, this wave mode always propagates at a slower speed than the quasi-longitudinal wave mode. Because it arrives in later portions of a waveform, it is usually not easy to identify. However, its amplitude changes dramatically when the polarization of either the source or the detecting transducer is changed. When the source and detector polarizations are aligned, the detected signal corresponding to the pure transverse wave mode reaches its maximum value. On the other hand, when the polarization of the source and receiving transducers are orthogonal to each other, the detected signal will be dramatically smaller. This provides the basis for a method of detecting the arrival of the pure transverse wave mode in a waveform. The polarizations of the source and detecting transducers are aligned and then made perpendicular to each other while the two resulting waveforms are superimposed, as shown in Fig. 9. The 'PT' wave arrival is taken as the point immediately prior to that portion of the waveform in which the largest excursion occurs when there is a 90° change in transducer polarization. The difference between the first-signal arrivals in the two polarizations is less than 0·25 μs. This difference may be the result of transducer coupling variability or aperture effects, or arise because the specimen material may not be

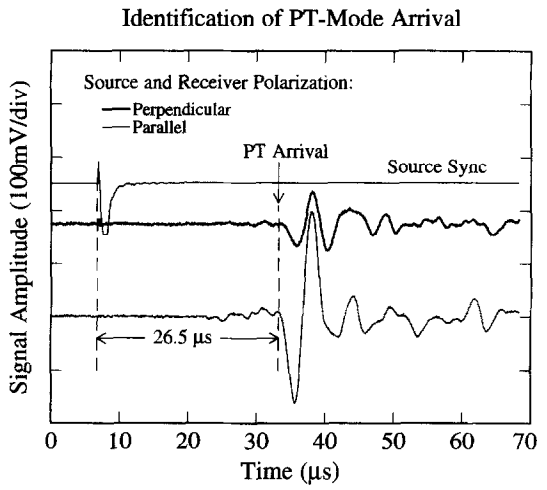

Fig. 9. Identification of the pure transverse 'PT' wave arrival at the epicenter.

perfectly transversely isotropic. By using the measured arrival of the 'PT' wave at the epicenter and eqn (3), we determine $C_{44} = 3{\cdot}75$ GPa. Then, from the arrival-time data of the 'PT' wave along directions whose angles were $\zeta = 47°$ and $\zeta = 60°$ and eqn (6), we find the value of C_{66} to be 3·81 GPa.

5.4 Measurement of the QL wave mode group velocity

In order to determine the elastic constants C_{11} and C_{13} according to the procedure outlined in Section 4, the group velocity data of the quasi-longitudinal (QL) wave mode group are needed along different directions specified by the angle ζ to the symmetry axis X_3. In order to obtain these data, PS/PR data were obtained by using the capillary fracture of a 0·25 mm diameter capillary as the source and a 1·3 mm aperture piezoelectric transducer as the detector. As before, the capillary was broken on a 70 μm thick, piezoelectric film of PVDF to generate a synchronization signal. The fracture of the capillary is a vertical load drop whose time dependence resembles a Heaviside step, which can be seen in Fig. 10. The arrival of the 'QL' wave mode in the detected signal is unambiguously identified as the first point from which the signal exceeds the background noise. The capillary fracture source was activated at a number of points along the top (smooth) surface of the composite plate. Two sample waveforms are shown in Fig. 10(a,b).

The group velocity data measured along different directions are shown in Fig. 11. Since the plate is expected to be transversely isotropic, V_g must be an even function of ζ, and the data are fit to a parabolic function. This is superimposed on the data in Fig. 11. The group velocity data as a function of direction are then converted into phase velocity data as a function of the wave normals, as described by eqns (8) and (9). The derived phase velocity data shown in Fig. 12 are fit to the theoretical curve of eqn (11), whereby the elastic constants C_{11} and C_{13} are adjusted until the best fit is found. The best fit is superimposed on the data in Fig. 12. The elastic constants are found to be $C_{11} = 20{\cdot}3$ GPa and $C_{13} = 8{\cdot}10$ GPa. Finally, the remaining elastic constant, C_{12}, is found from eqn (2)to be 3·81 GPa.

In assuming that the thick composite plate can be represented as an elastically transversely isotropic material, we have thus obtained the matrix of elastic constants listed in Table 2.

6 CONCLUSIONS

We have described in this paper the application of the ultrasonic point-source/point-receiver measurement technique to non-destructively determine the matrix of

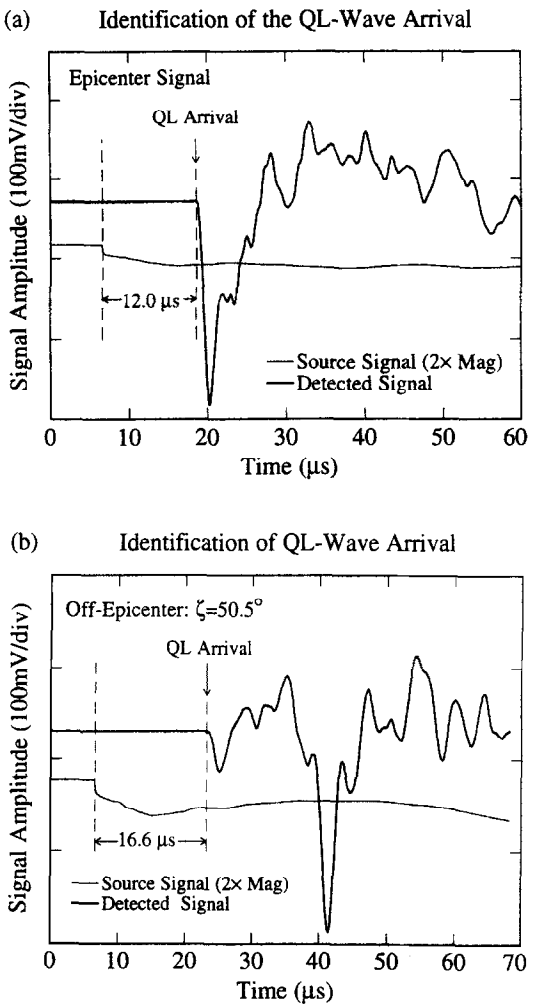

Fig. 10. Identification of the quasi-longitudinal wave arrival: (a) epicenter; (b) off-epicenter, $\zeta = 50{\cdot}5°$.

elastic stiffness constants of a thick composite plate. Knowledge of these constants for a fabricated material is essential for design purposes, but monitoring them

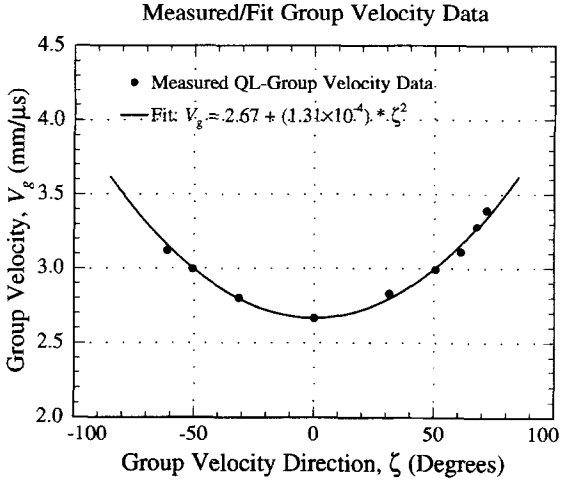

Fig. 11. Measured group velocity data and the polynomial fit to the data.

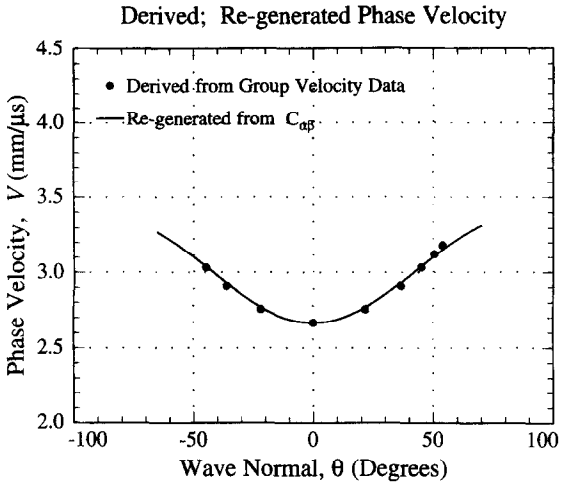

Fig. 12. Phase velocity derived from measured group velocity data, and phase velocity (solid line) as a function of wave normal computed from the recovered elastic constants and eqn (11).

in use may provide an indication of evolving microstructural characteristics of a thick composite as well as the presence and extent of damage. The determination requires no cutting or other preparation of the test structure. The procedure is based on measurement of PS/PR wave arrival-time data and hence the group velocities as a function of propagation directions in a specimen. These data are first used to identify the anisotropy of the specimen, and then measurements along specific directions permit recovery of a subset of the elastic stiffness matrix. Finally, measurements along arbitrary angles to the axis of transverse isotropy are used to determine the phase velocities and directions of wave normal and hence the remaining elastic stiffness constants. The simplicity of such measurements and the robustness of the inversion procedure make this approach useful for determining the stiffness characteristics of thick composite materials *in situ*.

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Table 2. Recovered elastic stiffness constants of a thick composite plate

$$C_{\alpha\beta} = \begin{bmatrix} 20{\cdot}3 & 12{\cdot}7 & 8{\cdot}10 & 0 & 0 & 0 \\ 12{\cdot}7 & 20{\cdot}3 & 8{\cdot}10 & 0 & 0 & 0 \\ 8{\cdot}10 & 8{\cdot}10 & 12{\cdot}5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3{\cdot}75 & 3{\cdot}75 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3{\cdot}81 \end{bmatrix} \text{(GPa)}$$

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