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Determination of the elastic constants of anisotropic materials using laser-generated ultrasonic signals

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This paper presents the solution of the materials characterization problem in which the elastic constants of an anisotropic material are determined from ultrasonic wavespeed measurements made in nonprincipal directions of a specimen. The ultrasonic waves were generated via the *point-source/point-receiver* technique using a pulsed laser as a source and a miniature, point-like transducer as a receiver. Data were acquired during a scan of the source along one of the principal acoustic axes of symmetry of the material. In each waveform the arrivals of the quasi-longitudinal and the two quasi-shear bulk modes were measured and the elastic constants of the material were then recovered using an optimization algorithm. Experimental results are presented for a transversely isotropic, unidirectional fiberglass/polyester and a single crystal specimen of silicon. It was found that the nonlinear fit between the measured and the recovered longitudinal slowness values is excellent. Some discrepancies are observed in the data for the two shear modes. These are shown to be related to the complexity of the detected signals.

I. INTRODUCTION

The determination of the elastic constants of a material from wavespeed measurements is a well-known procedure which has numerous applications in the field of materials characterization, cf. Ref. 1. This determination requires the solution of two problems. The first is related to the ultrasonic measurement technique itself. That is, the selection of the source and receiver, their coupling to the test specimen and the determination of the speed of propagation of particular elastic wave modes. Secondly, a robust processing algorithm is required which can be used to obtain a solution to the materials characterization problem, and, in particular, the procedure by which the elastic constants of the material are recovered from the measured wavespeeds. Not surprisingly, this second step becomes increasingly complex when one deals with low symmetry systems in which the number of independent elastic constants is large.[2]

An analysis of the propagation of elastic waves through an anisotropic medium results in the well-known Christoffel's equation whose characteristic equation is given by

$$\det|c_{ijkl}n_jn_k - \rho v^2\delta_{il}| = \Omega(v,\hat{\mathbf{n}}) = 0. \tag{1}$$

In this equation c_{ijkl} are the elastic constants of the material. The direction cosines of the wave propagation direction are specified by n_j and n_k, ρ is the density of the material, v are the phase velocities of the three bulk wave modes propagating through the material and δ_{il} is the Kronecker delta. Equation (1) is a cubic equation in terms of v^2 and the elastic constants c_{ijkl}. Using wavespeed data measured in particular propagation directions, this equation can be inverted to determine the elastic constants of the material. However, this inversion is, in general, complicated, particularly for materials possessing low orders of elastic symmetry. In the conventional procedure, the wavespeed measurements are made along specific directions for which the resulting equations simplify so that the inversion of the wavespeed data is easily accomplished to directly recover the elastic constants of the material. Unfortunately, this requires that the measurements be made along a number of specific crystallographic directions in the material and this can only be realized by cutting a number of particularly oriented specimens from it. Although this is time consuming and not always possible, it is the basis of the procedure most often used.

The algorithms by which wavespeed data measured along arbitrary propagation directions can be inverted to determine the matrix of elastic constants of a material include a number of different numerical methods. Some of these are based on perturbation techniques or series expansion methods.[3,4] However, these are inherently approximate as opposed to the more rigorous approaches which involve, for instance, the invariants of the Christoffel matrix.[5,6] An alternative method with no approximations has recently been proposed, whose principal application is for the characterization of synthetic and natural anisotropic materials.[7]

There is a diversity of techniques by which elastic waves can be generated and detected in a material. A review of these has been given in Ref. 8. Although piezoelectric transducers continue to be used for most applications, the advantages of noncontact sources and receivers has led

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to their increasing use. The use of pulsed lasers to generate elastic waves in solids by thermoelastic or ablative processes has been reviewed.[9,10] Its application to materials characterization measurements is becoming more widespread. Particulars of the laser-based ultrasonic technique used to generate the ultrasonic signals in the present study are given in the next section.

The selection of the sensor used to detect the ultrasonic signals must consider the requirements of detection of particular wave modes and a minimization of temporal and spatial convolution effects arising from utilizing a transducer of finite aperture. Also minimized must be the transducer loading effects and the associated signal modifications which can result in inaccuracies in the determination of the signal arrival times. The use of a laser-interferometric based point detector for ultrasonic wave arrival time measurements which overcomes many of the limitations of other transducers has been demonstrated.[11] Other suitable detectors of ultrasound, including those based on piezoelectric, electromagnetic and electrostatic principles have been summarized in Ref. 8.

The measurement of the phase velocities appearing in Eq. (1) for each of the bulk elastic wave modes is, in general, not trivial. In order to avoid multi-path effects, one most often utilizes transient excitations. The wavespeed measurement techniques which can be used with repetitive signals have been reviewed in Ref. 1. However, these are generally not useful when the excitations are single-shot or low repetition rate excitations. Instead, one must rely on arrival time measurements directly from the digitized waveforms, using a cross-correlation technique[12] or an analysis of the Fourier phase function of a particular wave arrival to determine the dispersion relation, $k(\omega)$, of the wave mode.[13–15] Once this is known, the *phase*, $v(\omega)$, or *group* velocity, $U(\omega)$, of the wave can be evaluated according to their definitions

$$v(\omega)=\frac{\omega}{k} \quad U(\omega)=\frac{\partial\omega}{\partial k}. \tag{2}$$

The group velocity can be found from the phase velocity

$$U(\omega)=v+k\frac{\partial v}{\partial k}=-\frac{\nabla_k\Omega}{\partial\Omega/\partial\omega} \tag{3}$$

where $\Omega(v,\hat{\mathbf{n}})$ is the function defined in Eq. (1). The gradient of Ω is taken with respect to the variables $k_x,k_y,k_z\equiv(n_x,n_y,n_z)k$. It is only possible in special circumstances to find the phase velocity from measurements of the group velocity. The difference between the phase velocity and the group velocity is related to the curvature of the former with respect to the direction cosines, n_x, n_y and n_z. When the curvature changes its sign, cusps, which are characteristic of the symmetry of the material, are seen in the group velocity.[16]

In the work described here, the arrival times were measured directly on the digitized waveforms. Although the corresponding wavespeeds did not correspond to the phase velocities, it will be seen from the results obtained, that the measured wavespeeds were sufficiently close to the phase velocities to permit utilizing the inversion scheme to be described. An algorithm for processing group velocity data was recently developed[17] and its application to determine the elastic constants of single crystal specimens of Si has been demonstrated.

Recent materials characterization applications utilizing laser-generated, laser detected ultrasonic signals have included the detection of flaws,[18] the characterization of material microstructures,[19] the determination of the elastic constants of isotropic solids at room temperature,[20] in industrial environments and at elevated temperatures,[18] and of composite materials.[21,22] However, little work has been completed which is related to the detailed characterization of the anisotropy of composite materials.[23]

II. MEASUREMENT TECHNIQUE

A. Measurement system

The development of the point-source/point-receiver, (e.g., PS/PR), measurement technique has provided a powerful, new tool which is facilitating simultaneous longitudinal and shear wave ultrasonic measurements in highly absorptive materials and irregularly shaped specimens.[13,14] If a well-characterized, small aperture, broadband source and receiver are used, and this is coupled with knowledge of the features of the propagation of transient elastic pulses through a bonded structure, one can process the detected ultrasonic signals to recover the true characteristics of the wave propagation in a specimen. However, even when uncalibrated point sensors are used, the technique facilitates wavespeed measurements in materials.[24]

When compared to conventional ultrasonic measurements based on finite aperture, contact transducer, laser-induced ultrasound exhibit several distinct advantages. These include: (1) Because of the boundary conditions at the surface, the laser/material thermoelastic interaction generates, in all but a few special situations, the three bulk wave modes in a specimen as predicted by theory; (2) The spherical waves generated by particular point sources exhibit a radiation pattern which permits the simultaneous propagation of waves in many directions; (3) Ideally, the point-source/point-receiver features of the system precisely define the angular characteristics of the direction of wave propagation not requiring diffraction corrections; (4) Because of the attainable source size, waves in small specimens of volume less than 1 cm^3 can be easily generated.

Additionally, the laser-based thermoelastic ultrasonic point source has a number of distinct advantages when it is employed in an ultrasonic system in which wavespeeds are to be measured. Its operating characteristics have been extensively investigated, cf. Refs. 25, 26. It is a broadband, fast-risetime (few ns) repetitive and a noncontact source which is readily amenable to operation in a scanning mode. When it is operated in the thermoelastic regime at low power levels and with no "constraining" layer in the target region, the source can be represented as crossed force dipoles, on the surface of the specimen. Such a source results in the generation of a high-amplitude bulk shear wave at receiver points in the near field of the source. Alternatively, at higher power levels and/or when there is a constraining layer in the target region, the laser source operates in the "ablative" regime, which can be represented as a force normal to the surface of the specimen. Such a source generates high-amplitude bulk longitudinal waves at receiver points in the near field of the source. The radiated fields of laser sources operating in the thermoelastic and ablative regimes are somewhat complementary. In the former, at large angles to the normal, the amplitudes of the longitudinal waves are large but the shear waves are small; when the source is ablative, these characteristics are observed at small angles to the normal.[9,10] When the laser source is operating in an intermediate regime, longitudinal and shear waves of reasonably uniform amplitudes will be radiated in all directions from the source point. The laser used to make the measurements reported in this paper was a Nd:YAG laser, generating pulses 2.5–7 ns in duration operating either in a single-shot mode or at a repetition rate of 1 Hz with an output pulse energy ranging typically from 100 to 275 mJ per pulse at a wavelength of 532 nm. The optical beam striking the specimen was focused to have a spot size of approximately 10 μm.

Ideally, one might use a laser-interferometer as a point detector of the ultrasonic signals.[11] Since none such was available for this study, the results presented here were obtained either with a miniature, broadband piezoelectric transducer whose aperture was 1.3 mm or a miniature capacitive transducer of 3.0 mm aperture. The fabrication details and operating characteristics of the latter have been described in Ref. 27. The transducer signals were amplified by 40 dB and input to a 10-bit waveform recorder sampling at 60 MHz which was connected to a minicomputer for subsequent signal recording and processing.

Regardless of the particular source and detecting transducer used, a precise triggering of the waveform digitizer is essential. This task is accomplished by using an external synchronization pulse provided by a photodiode mounted close to the oscillator of the laser. There is a systematic delay arising from the propagation of the light through the optical system. This shift is of order 10 ns which has been neglected because the waveforms were digitized at a sampling period of 16.7 ns and the measured time-of-flights are typically in the range of a few μs.

The minicomputer also controlled the scanning system whereby the sample with the transducer was scanned in the incident laser beam. To permit making wavespeed measurements at various propagation angles, a one-dimentional, iso-angular transmission scan, shown schematically in Fig. 1, was made along the principal acoustical axes of each specimen. By acquiring the wavespeed data at iso-angular increments, one gives an equal weight to the various angular sectors of the slowness surface.

B. Specimens

The measurements were made on specimens of a synthetic composite material as well as a single crystal specimen of Silicon. The composite specimens were flat plates of a uni-directional fiberglass/polyester resin which can be expected to be elastically transversely isotropic. The specimen dimensions were 101.6×101.6×6.35 mm thick with

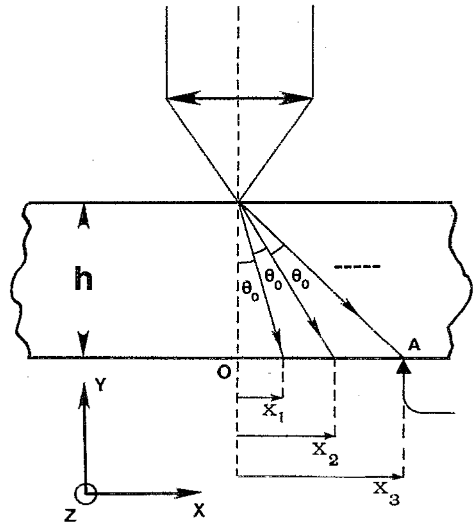


FIG. 1. Iso-angular scanning geometry.

the 5.65 mm thick, central core of the material consisting of glass fibers oriented approximately uni-directionally. This core was reinforced on each surface by a 0.35-mm thin layer whose orientation is mainly orthogonal to the direction of the fibers. The purpose of the outer layers is to transmit shear stresses and to insure the cohesion of the plate. A macrograph of the specimen material is shown in Fig. 2 in which the outer reinforcement layer is clearly visible. The samples were manually polished with a fine mesh, mechanical grinder to obtain smooth surfaces. In order to permit use of a capacitive transducer, a thin layer of conductive paint normally used for printed circuit applications was painted on one surface.

The second group of specimens were disks of Si of ⟨100⟩ orientation which were 76.2 mm in diam and 9.982 mm thick. In order to realize a higher source strength, the

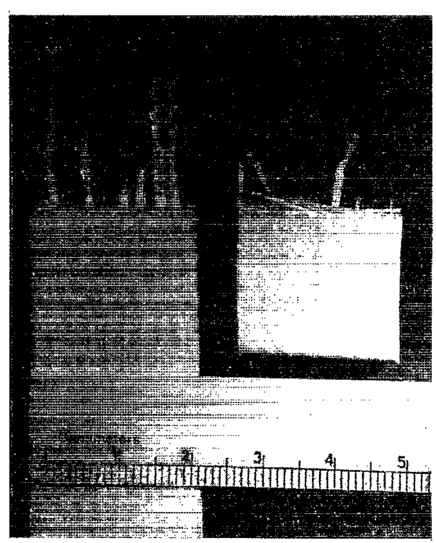


FIG. 2. General aspect of a specimen of the fiberglass/polyester resin, uni-directional composite material.

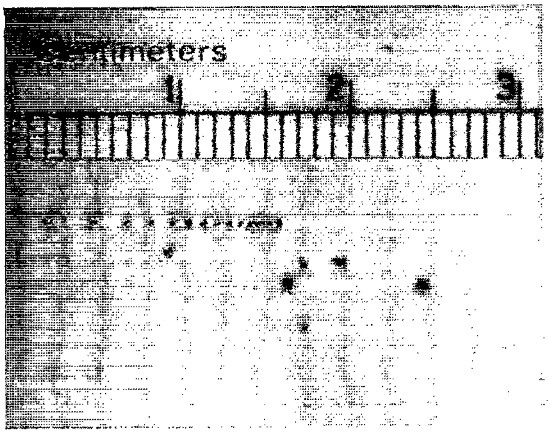


FIG. 3. Detailed view of the trace of the 1D iso-scan on the surface of the sample shown in Fig. 2.

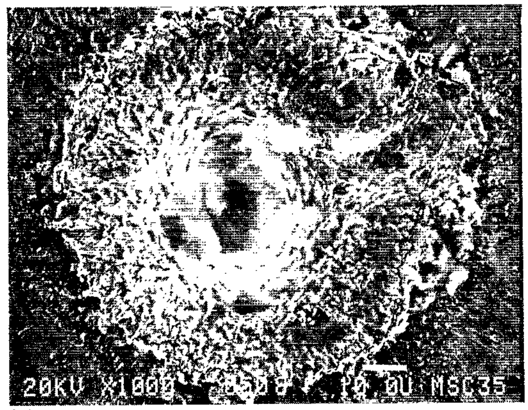
front surface of these were coated with a 5000 Å layer of aluminum. The linear thermal expansion of Si at room temperature is about 2.5×10^{-6}/K. A much higher source strength can be realized by coating the Si specimen with a thin film of aluminum whose coefficient of expansion is about ten times greater than that of Si.[10] Further, the aluminum coating will likely act as a "constraining" layer resulting in an additional enhancement of the source strength (cf. Ref. 10). A thin film of platinum was sputtered on the back surface of the Si specimen to obtain a conductive surface for the capacitive detector.

Even though the laser was operated at a power level such that the source of ultrasound was principally thermoelastic, it was evident that some superficial damage did occur on the front surface of the specimens. The damage resulting from one iso-angular transmission scan is shown in Fig. 3. The damage generated on the ⟨100⟩ surface of a single crystal of tungsten is shown in Fig. 4. We observed that the damage observed on the surface of the Si specimen was far less. It is clear from these observations that the sound generation occurred in the intermediate regime between thermoelastic and ablative laser-material interactions. The exact operating regime could be altered by changing the output power of the laser and/or the size of the source and observing the corresponding generated ultrasonic signals.

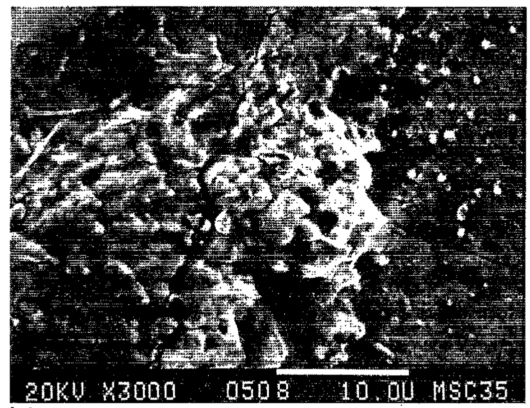
C. Data acquisition

Wavespeed measurements of the three quasi-bulk modes were made in the composite material over the principal planes (1,2) and (1,3)—where the index 3 is coincident with the fiber orientation. For the silicon single crystal, the elastic waves were generated on the face coinciding with the cubic plane. Iso-angular scans were taken along the fibers (i.e., axis 3) and along the in-plane perpendicular axis (axis 2) for the composite, and along the ⟨010⟩ or ⟨001⟩ directions in the {100} plane of a cubic single crystal.

Figure 5(a) is a characteristic example of the waveform obtained at normal incidence for the fiberglass/



(a)



(b)

FIG. 4. (a) SEM viewgraphs of laser-induced damage in a single crystal of tungsten ⟨100⟩. (b) Enlargement of (a).

polyester plate with a 1.3-mm-diam piezoelectric receiver. Although this signal is complex, a number of features can be identified. These are indicated in the figure where 1P, S_{FT}, and S_{ST} correspond to the arrivals of the longitudinal, fast transverse and slow transverse bulk modes, respectively. The correct identification of the two transverse wave modes is no trivial matter and some additional information is usually needed for this purpose. One means of obtaining this is to replace the point piezoelectric transducer with a broadband capacitive displacement transducer of similar aperture. Because of the high fidelity of the latter, the characteristic, temporal features in the waveforms are more easily identifiable. The capacitive transducer cannot be used in all applications because of its reduced sensitivity when compared to the piezoelectric sensors. Figure 5(b) is the waveform detected with a 3-mm-diam capacitive transducer when the source was located directly on the opposite side of the plate specimen. The comparison of the two waveforms permits positive identification of the two weak signals corresponding to the fast and slow transverse waves, S_{FT} and S_{ST}. Other, complementary experimental evidence for the correct identification of the transverse wave modes can be obtained by using contact shear piezo-

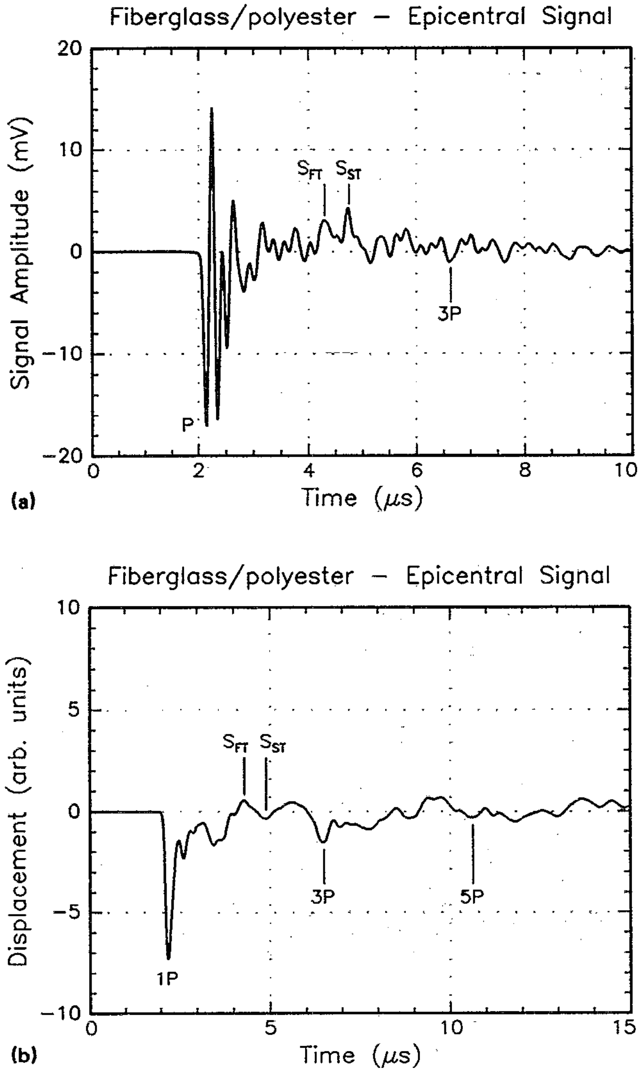


FIG. 5. Detected waveforms at normal incidence in the uni-directional composite material. The longitudinal wave signal is denoted by P and the fast and slow transverse wave modes are denoted by S_{FT} and S_{ST}, respectively. (a) Using a 1.3-mm-diam piezoelectric transducer. (b) Using a 3.0-mm-diam capacitive transducer.

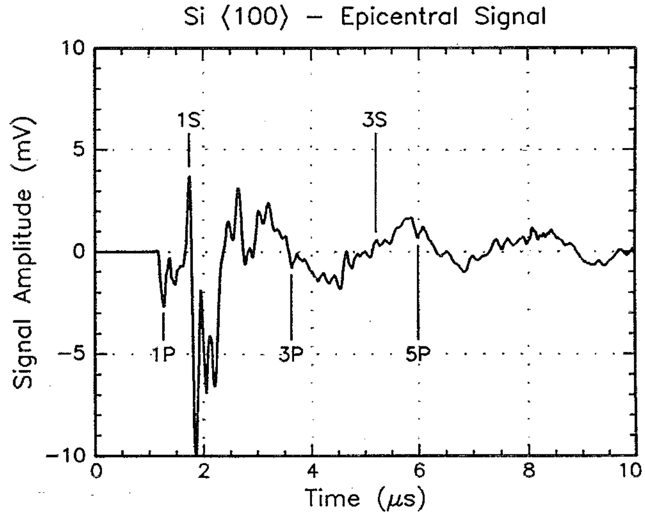


FIG. 6. Waveform at normal incidence in the Si ⟨100⟩ single crystal obtained with a 1.3-mm-diam piezoelectric transducer.

electric transducers to detect the signals. As Fig. 6 illustrates, positive identification of the shear wave arrival in a cubic single crystal specimen is possible even when a piezoelectric sensor is used, provided that the signal detected at normal incidence is measured. In this case, the shear modes are degenerate and pure so that their arrival is readily identified by the sharp discontinuity labelled 1S in the figure. In contrast to the composite material, the two first reflections of the pressure wave in the single crystal (labelled 3P and 5P) and the first reflection of the shear wave (labelled 3S) are also identified in the waveform. When the transmission angle differs from zero, the identification of the various modes can be carried out inductively by comparing the waveforms at two successive transmission angles, whereby the modes have been identified in one of the signals with the initialization of the mode identification process being made at normal incidence. This scheme, which has been implemented as a manual procedure up to now, could be automated in the future with an appropriate signal processing algorithm.

III. WAVESPEED DATA INVERSION

When the principal acoustical axes of the material are not known, the propagation of the elastic waves is, in general, in arbitrary, nonprincipal planes. A general numerical method has recently been proposed which optimally solves this problem.[7] Fortunately, in many practical cases, the orientation of the principal axes of symmetry in a specimen is known *a priori*. In that case, it is possible to propagate the elastic waves in one of the principal planes of the material and thus the cubic equation resulting from Eq. (1) simplifies to the product of a trivial linear term and a quadratic expression. Simple analytical functions relating the wavespeeds to the elastic constants then result.[28] For waves propagating in the plane containing the axes of the fibers of a transversely isotropic composite material, these expressions are:

$$\text{Pure shear mode:}\quad \frac{1}{v_S}=\sqrt{\frac{\rho}{C_{66}\sin^2\theta+C_{44}\cos^2\theta}}, \tag{4}$$

$$\text{Quasi-shear mode:}\quad \frac{1}{v_{qS}}=\sqrt{\frac{2\rho}{(a-b)}}, \tag{5}$$

and

$$\text{Quasi-longitudinal mode:}\quad \frac{1}{v_{qP}}=\sqrt{\frac{2\rho}{(a+b)}}. \tag{6}$$

In the above equations

$$a\equiv C_{11}\sin^2\theta+C_{33}\cos^2\theta+C_{44}$$

and

$$b \equiv \{[(C_{11}-C_{44})\sin^2\theta+(C_{44}-C_{33})\cos^2\theta]^2 + (C_{13}+C_{44})^2\sin^2 2\theta\}^{1/2}.$$

With nonlinear, least-square procedures the above equations can be readily inverted to optimally recover the elastic constants of the material from wavespeed data.[29] This has been demonstrated for materials of orthorhombic symmetry and for transversely isotropic materials as a subset.[30] That is, for a transversely isotropic material, the plane (1,2) is isotropic and any plane containing the 3-axis is equivalent. These algorithms have been applied here with only slight modifications. It is noted that the cubic symmetry is a subset of the tetragonal symmetry (i.e., the Hermann–Manguin classes *4mm, 422, $4\bar{2}m$, 4/mmm*) with the additional identities:

$$C_{33}=C_{11},\quad C_{66}=C_{44},\quad \text{and } C_{13}=C_{12}. \qquad (7)$$

Alternatively, one begins by assuming that the material under test is elastically tetragonal, determine its elastic constants, verify that the above identities hold and subsequently deduce that the material is, in fact, cubic.

IV. RESULTS

The uni-directional composite material whose fibers are aligned in the plane of the specimen (in direction 3) was scanned in two principal directions (2 and 3, respectively). In the first case, the directions of wave propagation lie in the (1,2) plane of the specimen. For that case, the wavespeeds are isotropic and can be computed by using the angle $\theta=90°$ in Eqs. (4)–(6). It follows that the measured wavespeed data can be directly inverted to recover the elastic constants: C_{11}, C_{44}, and C_{66}. The value of C_{12} can then be computed using the relation: $C_{12} = C_{11} - 2C_{66}$. The results obtained on the fiberglass/polyester specimen are shown in Fig. 7(a). The data points correspond to the measured slowness values and the curves are the slowness curves computed from the recovered elastic constants.

For waves propagating in the (1,3) plane of the composite, Eqs. (4)–(6) also apply. The results obtained for such a scan in the fiberglass/polyester specimen are shown in Fig. 7(b). It is seen in both Figs. 7(a) and 7(b) that the agreement between the measured and the reconstructed longitudinal wave slowness values is excellent. By comparison, the agreement for the two shear modes at several orientations is only modest, principally because of the impreciseness in finding the wave arrival in the detected signals. An additional uncertainty results from the 16.7 ns sampling rate used to record the waveforms and the resulting impreciseness in determining the wave arrivals. Table I is a compilation of the recovered elastic constants.

The uncertainties shown in Table I were obtained using a procedure similar to that described in an earlier paper.[7] In the present case, the deviations between the slowness values computed from the measured data of arrival time, propagation path and direction are compared with those predicted from the recovered elastic constants. A set of simulated input data is then constructed which is perturbed by this deviation. This data is then used with the inversion algorithm to recover perturbed elastic constants. Repeating this a number of times according to the desired precision, the mean values and standard deviations of the elastic constants can be determined. The standard deviations so found, expressed in terms of a coefficient of variation, are expected to correspond to the accuracy of the inversion procedure. It is these variations that are expressed in parenthesis in Table I. It is recognized that this approach only permits an estimation of the random errors

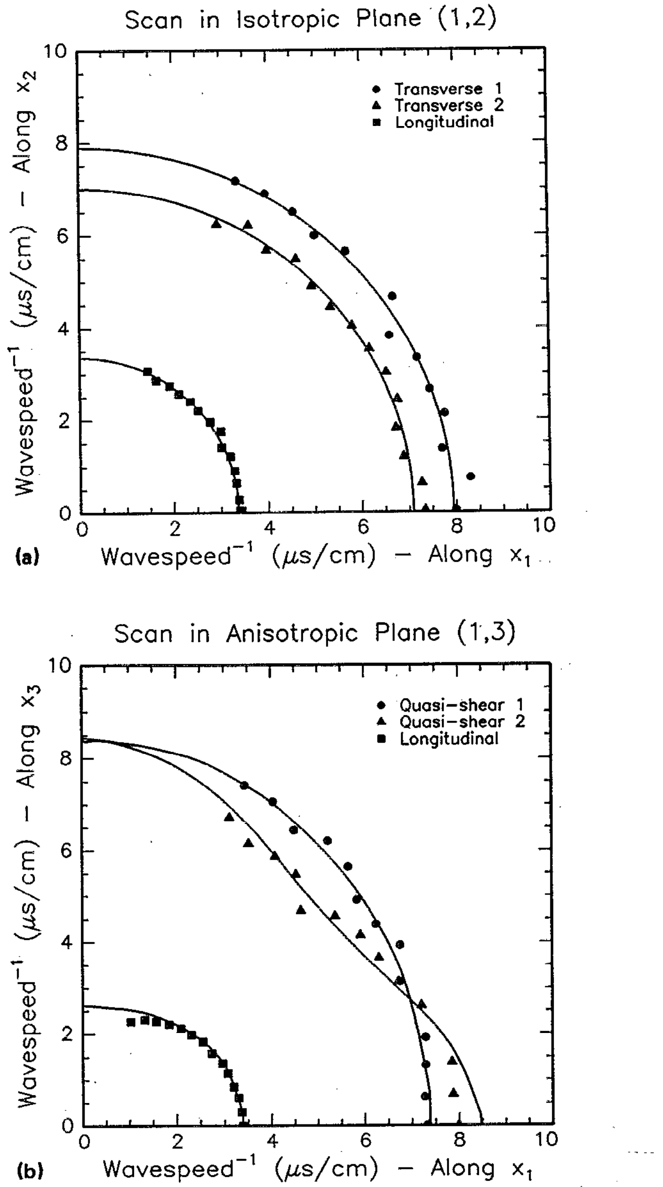


FIG. 7. Experimental (●; ▲; ■) and recovered (—) slownesses of the uni-directional composite ($\rho = 1.90$ g/cm^3). (a) In the isotropic plane (1,2). (b) In the anisotropic plane (1,3).

TABLE I. Elastic constants of uni-directional fiberglass/polyester [GPa].

$C_{11}=16.72$ (±0.6%)	$C_{33}=28.41$ (±1.4%)
$C_{66}=3.69$ (±0.8%)	$C_{44}=2.85$ (±6%)
$C_{12}=9.34$ (±0.8%)	$C_{13}=12.76$ (±10%)

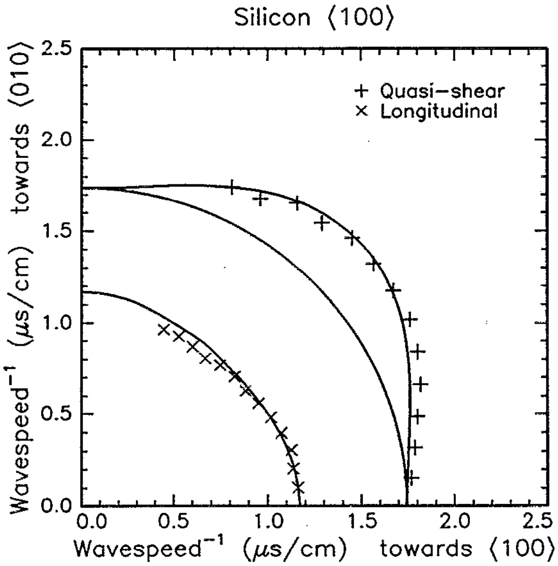


FIG. 8. Experimental (+ ; ×) and recovered (—) slownesses in the ⟨100⟩ face of a cube of single crystal Si.

involved in the inversion procedure; systematic errors are much more difficult to deal with and these have not been considered.

The measurements made over plane (1,3) exhibit the interesting feature of the intersection between the pure and the quasi-shear modes. This occurs at the angle specified by

$$\theta_I = \cot^{-1}\sqrt{\frac{(C_{13}+C_{44})^2-(C_{11}-C_{66})(C_{33}-C_{44})}{(C_{44}-C_{66})(C_{11}-C_{66})}}, \tag{8}$$

which is approximately 21°.

The measured and recovered slowness values of waves in the single crystal specimen of Si are shown in Fig. 8. Although there is some disagreement between these values for the quasi-longitudinal mode, the recovered elastic constants agree reasonably well with the published values for this material.[17,31] The cubic elastic constants determined in this study are listed in Table II and, for comparison, those determined using group velocity data[17] and a conventional ultrasonic technique.[31] It is seen that the elastic constants determined in this study agree with the previously measured values to within 3%.

In making these measurements, the arrivals corresponding to the fast-transverse (FT) signals could not be clearly identified in the waveforms and for this reason, these data do not appear in Fig. 8. Although the origin of this undetectability is not yet fully understood, a possible

TABLE II. Elastic constants of Si [GPa].

Elastic constant	This work [GPa]	Group velocity data [GPa][17]	Published value[31]
C_{11}	170.6	165.1	165.7
C_{44}	77.4	80.2	79.6
C_{12}	62.5	65.0	63.9

explanation may be obtained from physical considerations. As mentioned in Sec. II.A., the laser source mechanisms can be modelled in terms of forces possessing an axial symmetry about the normal to the specimen surface.[10] When the wave propagation direction in the material is arbitrary, the waves whose particle displacements are out of the sagittal plane (defined by the propagation direction and normal to the specimen surface) will have small amplitudes. In other words, the dominant radiated waves possess polarizations corresponding to longitudinal (P) and shear-vertical (SV) type, and minimally of shear-horizontal (SH) type. This has been confirmed by recent, detailed measurements of laser-generated sound fields in anisotropic solids.[32] For the propagation directions considered in this work, the fast-transverse waves are SH in character while the slow-transverse (ST) modes are principally of SV type. Hence, a laser source can be expected to mainly radiate P and ST waves. Further, since the sensors used to make the measurements favor the detection of motions normal to the specimen surface, it is not surprising that only signals corresponding to the P and S_{ST} wave arrivals are clearly identifyable in the detected signals.

It should be emphasized that the measurement method described here is very general. One extremely favorable feature of the method is the fact that the iso-scan can be done on very small samples. As Fig. 3 shows, the scan for the twelve data points, corresponding to an angular range of 55° in transmission, is realized over only 9 mm of the sample. In addition, by reducing the thickness of the samples, this value can be decreased provided that the temporal resolution in the waveforms remains acceptable. A limit to that is related to the aperture of the receiving sensor as well as the waveform digitization rate. A point-like optical probe would be clearly advantageous for work with minute specimens.

V. CONCLUSIONS

A procedure has been described by which a generalized characterization of the elastic properties of an anisotropic material can be made. The procedure utilizes a pulsed Nd:YAG laser as an ultrasonic source and a miniature piezoelectric or capacitive transducer as a detector. From this work, the following conclusions can be drawn:

(1) The three bulk wave modes of propagation are generally observed in various planes of propagation ranging from normal incidence to large angular regions. This contrasts with conventional ultrasonic measurements, e.g., immersion systems, in which one generaly has access to only two wave modes over a far smaller range of directions.

(2) The recovery of the elastic constants of an unidirectional fiberglass/polyester composite was demonstrated. It was shown that this composite can be appropriately modelled as being elastically transversely isotropic. The fit between the measured and reconstructed slowness values is generally good and excellent for the quasi-longitudinal mode.

(3) The recovered elastic constants of a single crystal of Si of orientation ⟨100⟩ are in good agreement with pre-

viously published data and the fit between the measured and reconstructed slowness values is acceptable.

(4) An important element in the measurement procedure which has been described is that the data are acquired over only a small region of the sample, typically one to two thickness dimensions long. By using thin samples and achieving sufficient temporal resolution in the detected signals which can be obtained with a high-fidelity, point-like, noncontact optical probe, it should be possible to characterize very small specimens with this method.

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